Proposal for a Master Thesis:

Polynomial chaos for parabolic partial differential equations

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We consider the heat equation in one space dimension, i.e., a parabolic partial differential equation (PDE) of second order. We assume uncertainties in the corresponding heat conduction. Consequently, the coefficient of the heat conduction is replaced by a random variable. Now the stochastic model has to be resolved by some numerical method. The technique of the polynomial chaos yields a larger coupled system of parabolic PDEs. The properties of the larger coupled system have to be analysed. Numerical methods to solve the system of PDEs shall be implemented. In particular, the stability of finite difference methods can be investigated in case of the larger coupled system. An open question is if step size restrictions, which result in explicit methods, will be the same or stronger or weaker than in case of the original PDE.

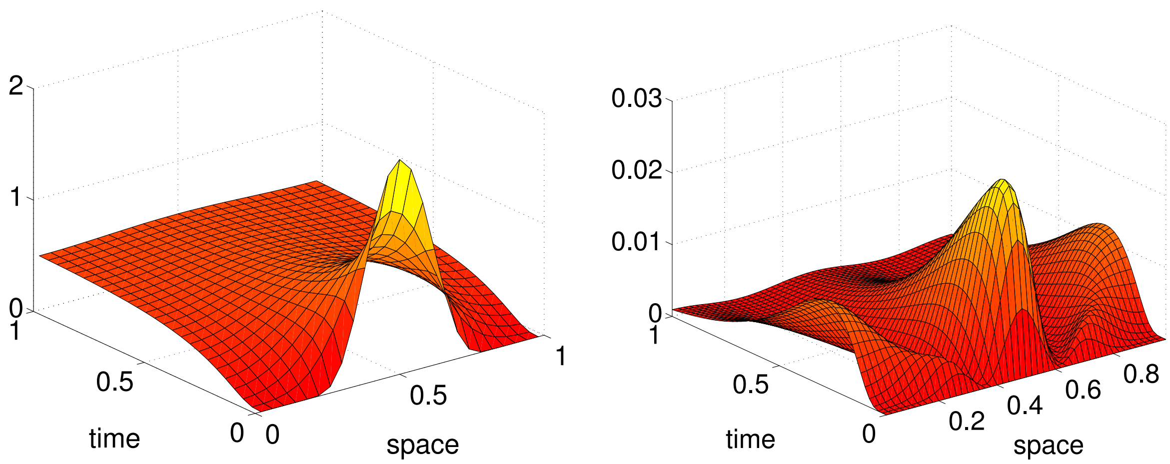

Figure 1: Expected values (left) and variances (right) for solution of heat equation with random heat conductance.

Description of tasks:

- introduction into polynomial chaos techniques
- modelling of heat conduction as random variable
- investigation of coupled system for heat equation
- stability analysis of finite difference methods for coupled system
- implementation of finite difference method for coupled system
- numerical simulation and visualisation of results

Alternatives/Extensions:

- modelling of heat conduction as space-dependent random process
- two or three dimensions in space
- other parabolic partial differential equations

Required Knowledge:

- Numerical Analysis and Simulation of PDEs
- basics in Stochastics

Literature:

- Ch. Großmann, H.-G. Roos, M. Stynes: Numerical Treatment of Partial Differential Equations. Springer, Berlin, 2007. (for parabolic equations)
- D. Xiu: Numerical Methods for Stochastic Computations. Princeton University Press, New Jersey, 2010. (for polynomial chaos)

Possible cooperations:

- Prof. Dr. Peter Rentrop (TU München)
- Dr. Utz Wever (Siemens AG, München)