Master Thesis Proposal:

Finite difference and spectral methods in financial engineering PDEs

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In financial engineering the pricing of derivatives is a major field of interest. The price dynamics arising here, can be represented by convection diffusion partial differential equations (PDEs). Typical examples are the Black Scholes equations

$$\frac{\partial u}{\partial \tau} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u}{\partial S^2} + rS\frac{\partial u}{\partial S} - ru$$

and its extensions, e.g. the SABR model or the Heston-model with stochastic volatility

$$\frac{\partial u}{\partial \tau} = \frac{1}{2}v^2 S^2 \frac{\partial^2 u}{\partial S^2} + \rho\sigma v S \frac{\partial^2 u}{\partial v \partial S} + \frac{1}{2}\sigma^2 v \frac{\partial^2 u}{\partial v^2} - ru + rS\frac{\partial u}{\partial S} + \left[\kappa(\theta - v) - \lambda(S, v, t,)\right]\frac{\partial u}{\partial v}.$$

There exists a large number of different techniques to solve these equations , such as standard Finite-Difference-Methods (FDM), Alternating-Direction-Implicit (ADI) [3, 2] or Locally-One-Dimensional (LOD) Splitting schemes. These finite difference schemes use local approximation to produce tridiagonal systems, which can be solved very efficiently in linear runtime. Whereas conversely in Spectral methods [4, 1] global basis functions are employed.

In this thesis both numerical approaches (local, global) shall be applied to option pricing problems. Each scheme shall be analysed in detail regarding its numerical properties, e.g. consistency and convergence, respectively. Furthermore the properties of the methods with respect to accuracy versus computational costs shall be compared.

Tasks

- study existing methods for financial problems
- analysis and implementation (e.g. Matlab, Python) of numerical schemes
- derivation of new numerical methods

References

[1] C. Canuto, M. Y. Hussaini, A. Quarteroni, and T. A. Zang. *Spectral Methods: Fundamentals in Single Domains*. Springer, 2011.

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[3] K. J. in't Hout and B. Welfert. Stability of ADI schemes applied to convection-diffusion equations with mixed derivative terms. *Applied Numerical Mathematics*, 57:19–35, 2007.

[4] L. N. Trefethen. *Spectral Methods in MatLab*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2000.